Reach System 1 Kinematic and Dynamic Properties

Blueprint Lab

Last Update: September 2019

1 Kinematic Properties

Link	d (mm)	$\boldsymbol{\theta}$	a (mm)	$\boldsymbol{\alpha}$
0	46.2	$\theta_0 + \pi$	20	$\pi/2$
1	0	$\theta_1 - \theta_a$	150.71	π
2	0	$\theta_2 - \theta_a$	20	$-\pi/2$
3	-180	$\theta_3 + \pi/2$	0	$\pi/2$
4	0	$-\pi/2$	0	0

Table 1: Standard DH Parameters for R5M with $\theta_a = \tan^{-1}\left(\frac{145.3}{40}\right)$

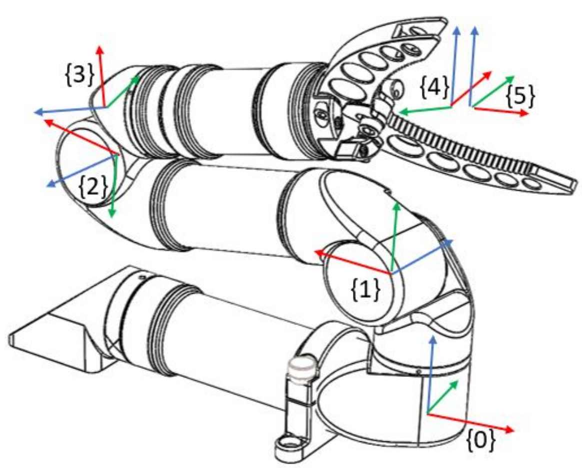

Figure 1: R5M joint frames (x,y,z)

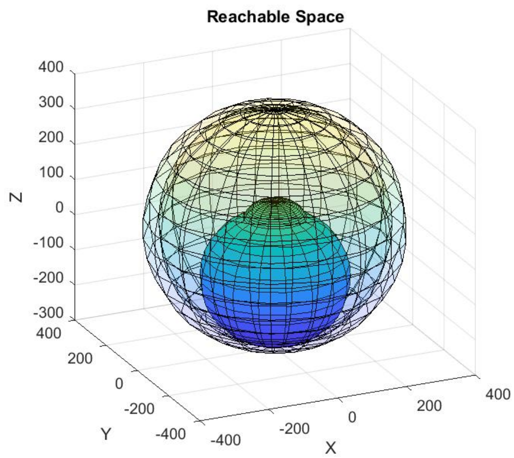

Figure 2: Reachable workspace without self collision showing inner and outer limits

1.1 Workspace

The outer reachable limits form a torus

$$(\sqrt{x^2+y^2}-a_0)^2+(z-d_0)^2 \leq (a_1+\sqrt{d_3^2+a_2^2})^2 \tag{1}$$

The inner reachable limit is the torus

$$(\sqrt{x^2+y^2}-a_0)^2+(z-d_0)^2 \geq ((39.94+a_2)^2+(145.3+d_3)^2) \tag{2}$$

The inner reachable limit with the arm in the downward position is the torus

$$(\sqrt{x^2+y^2}-a_0)^2+(z-d_0+145.3)^2 \geq (-d_3)^2 \tag{3}$$

These are shown in Figure 2.

1.2 Inverse Kinematics

From Figure 3 we can solve for the underarm solution

$$\theta_0 = tan2^1(\frac{y}{x})+\pi \tag{4}$$

$$R = \sqrt{x^2+y^2} \tag{5}$$

From Figure 4 we can solve for

$$l_1 = a_1 \tag{6}$$

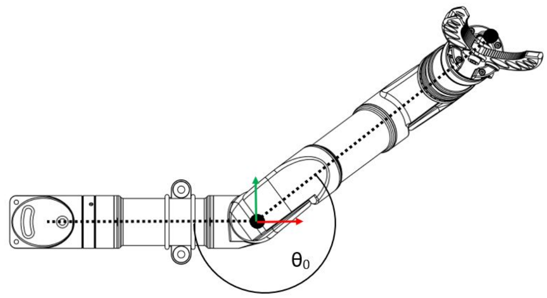

Figure 3: Calculation of θ_0

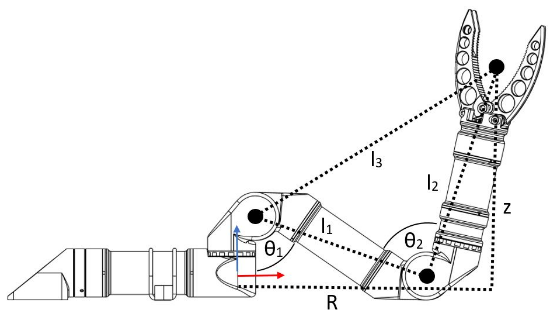

Figure 4: Calculation of θ_1 and θ_2

$$l_2 = \sqrt{a_2^2 + d_3^2} \tag{7}$$

$$l_3 = \sqrt{(R - a_0)^2 + (z - d_0)^2} \tag{8}$$

$$\theta_2 = cos^{-1}(\frac{l_1^2 + l_2^2 - l_3^2}{2l_1 l_2}) - sin^{-1}(\frac{2a_2}{l_1}) - sin^{-1}(\frac{a_2}{l_2}); \tag{9}$$

$$\theta_1 = \frac{\pi}{2} + tan2^{-1}(\frac{z - d_0}{R - a_0}) - cos^{-1}(\frac{l_1^2 + l_3^2 - l_2^2}{2l_1 l_3}) - sin^{-1}(\frac{2a_2}{l_1}) \tag{10}$$

Now we can also solve for the overarm solution from Figure 5

$$\theta_0 = tan2^1(\frac{y}{x}) \tag{11}$$

Finally from Figure 6

$$l_3 = \sqrt{(R + a_0)^2 + (z - d_0)^2} \tag{12}$$

$$\theta_2 = cos^{-1}(\frac{l_1^2 + l_2^2 - l_3^2}{2l_1 l_2}) - sin^{-1}(\frac{2a_2}{l_1}) - sin^{-1}(\frac{a_2}{l_2}); \tag{13}$$

$$\theta_1 = \frac{3\pi}{2} - tan2^{-1}(\frac{z - d_0}{R + a_0}) - cos^{-1}(\frac{l_1^2 + l_3^2 - l_2^2}{2l_1 l_3}) - sin^{-1}(\frac{2a_2}{l_1}) \tag{14}$$

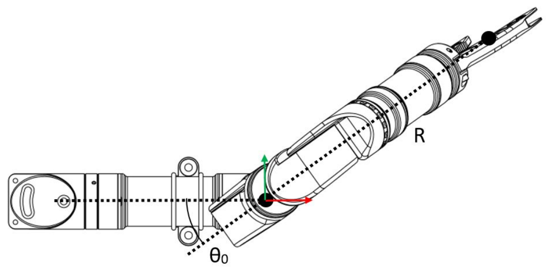

Figure 5: Calculation of θ_1 and θ_2

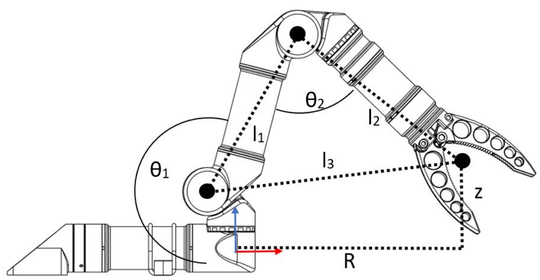

Figure 6: Calculation of θ_0

2 Inertial Properties

Link	Mass (kg)	COM (mm)	I ($kg.mm^2$)
0	0.341	$\begin{pmatrix} -75 & -6 & -3 \end{pmatrix}$	$\begin{pmatrix} 99 & 139 & 115 \\ 139 & 2920 & 3 \\ 115 & 3 & 2934 \end{pmatrix}$
1	0.194	$\begin{pmatrix} 5 & -1 & 16 \end{pmatrix}$	$\begin{pmatrix} 189 & 5 & 54 \\ 5 & 213 & 3 \\ 54 & 3 & 67 \end{pmatrix}$
2	0.429	$\begin{pmatrix} 73 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 87 & -76 & -10 \\ -76 & 3190 & 0 \\ -10 & 0 & 3213 \end{pmatrix}$
3	0.115	$\begin{pmatrix} 17 & -26 & -2 \end{pmatrix}$	$\begin{pmatrix} 120 & -61 & -1 \\ -61 & 62 & 0 \\ -1 & 0 & 156 \end{pmatrix}$
4	0.333	$\begin{pmatrix} 0 & 3 & -98 \end{pmatrix}$	$\begin{pmatrix} 3709 & 2 & -4 \\ 2 & 3734 & 0 \\ -4 & 0 & 79 \end{pmatrix}$

Table 2: Inertial properties for R5M

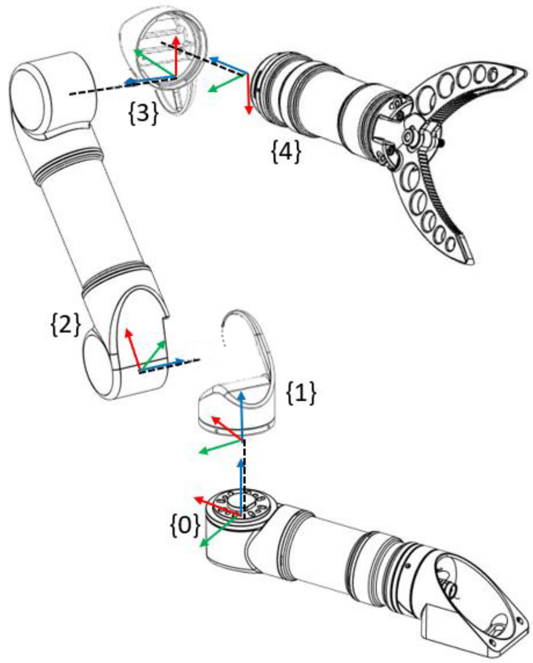

Figure 7: Inertial frames for each link (x,y,z)

3 Hydrodynamic Properties

3.1 Added Mass

We use standard equations for added mass assuming cylindrical sections with spherical ends.

Link	$(\boldsymbol{X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}})(\boldsymbol{kg})$	$(\boldsymbol{K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}})(\boldsymbol{kg.mm^2})$
0	$\begin{pmatrix} 0.017\rho & 0.189\rho & 0.189\rho \end{pmatrix}$	$\begin{pmatrix} 0 & 1414\rho & 1414\rho \end{pmatrix}$
1	$\begin{pmatrix} 0.032\rho & 0.032\rho & 0.017\rho \end{pmatrix}$	$\begin{pmatrix} 7\rho & 7\rho & 0 \end{pmatrix}$
2	$\begin{pmatrix} 0.017\rho & 0.201\rho & 0.201\rho \end{pmatrix}$	$\begin{pmatrix} 0 & 1716\rho & 1716\rho \end{pmatrix}$
3	$\begin{pmatrix} 0.032\rho & 0.017\rho & 0.032\rho \end{pmatrix}$	$\begin{pmatrix} 7\rho & 0 & 7\rho \end{pmatrix}$
4	$\begin{pmatrix} 0.226\rho & 0.226\rho & 0.017\rho \end{pmatrix}$	$\begin{pmatrix} 2443\rho & 2443\rho & 0 \end{pmatrix}$

Table 3: Added mass terms where $\rho \sim 1$ is the density in kg/L

3.2 Viscous Damping

We consider only quadratic drag assuming Reynolds numbers above the Stokes flow approximation at any significant velocity. We also consider only translational drag. We approximate the Reynolds number as

$$Re = \frac{\rho u L}{\mu} \sim \frac{10^3 * 0.1 * 0.1}{10^{-3}} \sim 10^{-4} \quad (15)$$

where ρ is the fluid density, u is the relative velocity, L is the characteristic length and μ is the dynamic viscosity, which gives a drag coefficient of $c_d \sim 0.5$. We now calculate the quadratic drag using

$$F_d = \frac{1}{2}\rho u^2 c_d A \quad (16)$$

where A is the cross sectional area.

The centre of drag is assumed to coincide with the centre of buoyancy

Link	$(X_{u\|u\|}, Y_{v\|v\|}, Z_{w\|w\|})$ $(N/\sqrt{ms^{-1}})$
0	$\begin{pmatrix} 0.3\rho & 1.5\rho & 1.5\rho \end{pmatrix}$
1	$\begin{pmatrix} 0.26\rho & 0.26\rho & 0.3\rho \end{pmatrix}$
2	$\begin{pmatrix} 0.3\rho & 1.6\rho & 1.6\rho \end{pmatrix}$
3	$\begin{pmatrix} 0.26\rho & 0.3\rho & 0.26\rho \end{pmatrix}$
4	$\begin{pmatrix} 1.8\rho & 1.8\rho & 0.3\rho \end{pmatrix}$

Table 4: Drag terms where $\rho \sim 1$ is the density in kg/L

3.3 Buoyancy

Link	**Volume** (L)	**COB** (mm)
0	0.202	$\begin{pmatrix} -75 & -6 & -3 \end{pmatrix}$
1	0.018	$\begin{pmatrix} -1 & -3 & 32 \end{pmatrix}$
2	0.203	$\begin{pmatrix} 73 & 0 & -2 \end{pmatrix}$
3	0.025	$\begin{pmatrix} 3 & 1 & -17 \end{pmatrix}$
4	0.155	$\begin{pmatrix} 0 & 3 & -98 \end{pmatrix}$

Table 5: Buoyancy terms with Centre of Buoyancy (COB)

4 Actuator Properties

$$torque = 90.6 * (current \pm 43.0) \tag{17}$$

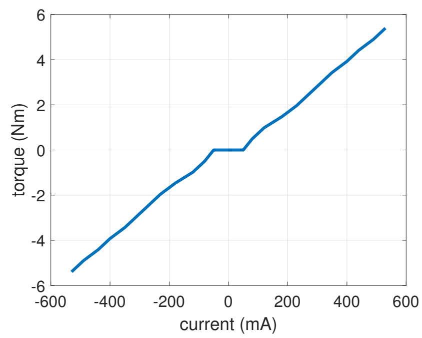

Figure 8: Plot of torque vs current for high torque joint